TEACHING OF FACTORING.

By E. M. Dow.

When new subjects seek admission to the curriculum on equal terms with their elder sisters (or on any terms, for that matter), they are told that their subject matter has not been organized, tested and tried in the crucible of time, and hence their educational value is uncertain at best. In the case of mathematics great minds have been concerned with its teaching for hundreds of years and thousands of texts have put before students the principles and methods of the science until we moderns surely have a right to expect under Darwin's law of survival of the fittest that the methods and processes of the present day are the best up to date if not indeed perfect. And the progressive adaptation of mathematics and its teaching to the spirit of the times has been due in part to the great geniuses whose names we respect and whose works are imperishable; and partly to the rank and file of mathematics teachers who by intention or good fortune have hit upon a device that works and have made this common property for their fellows. So today I wish, as a little one in mathematics and not even as the original discoverer of the device, to bring you a scheme for factoring which "works" very well. I am perhaps only the mouthpiece for my colleagues who use the scheme with even more deadly effect than I do.

In speaking on this topic of factoring I shall assume that the texts bring out with sufficient clearness how a polynomial is analyzed into its factors, that in our days of abounding vitality and confidence when we feel that we are really teaching we preface our work in factoring with the special theorems in multiplication and make our students realize that we are at the task of undoing work that resulted in the expression before us; that we teach carefully with many examples how a monomial factor is spotted and ferretted out, how binomials with two squares and a minus sign or two cubes or two odd powers

and a sign are treated, how a trinomial yields to the analyst's hammer and how more imposing arrays of terms are conquered. In other words I shall assume that we teach our classes sufficient algebraic "materia medica" so they will normally apply the correct dose to a given case of factoring or will use the scalpel skilfully when the nature of the lesion is apparent.

To follow out the medical analogy just suggested, a doctor must know not only what medicine to apply to a given disease but must also be able to decide what disease is present in a patient before him: his diagnosis precedes any application of healing remedies. When confronted with certain symptoms he must say to himself "Now this sore throat may be from tonsilitis, quinsy, diphtheria, etc., this fever from several sources, etc.," and each added symptom helps him to reach a correct decision on the exact nature and name of the disease. Then his training in dosage and pillage sends his fingers to a suitable bottle in his satchel. Isn't diagnosis the more difficult part to practise with security? And doesn't the medical school lay emphasis on both diagnosis and prescribing?

To take another analogy and look at troubles through the eyes of a wire chief in a telephone exchange. When a subscriber sends a complaint the first duty of the wire chief is to find out what sort of trouble is present, and that often takes rather extended search. He knows what to do for almost any form of short circuit, wet cord or crossed wires, loose connections or broken poles, but he doesn't get out tools for any one of them until he finds out what the trouble is and what tools he can use to advantage.

Again a salesman for either specialties or staples must know his catalogue pretty well in order to sell the goods his customer needs or wants. And if a lady asks to see some dress goods, the clerk must find out whether she means silk or broadcloth or calico and can then take down the right kind.

In all these cases I have tried to show that diagnosis or "sizing up" is fully as important as the determined action which follows or accompanies; that often the required action is brief compared to the study of conditions; and the instances cover a fairly wide range of experience. Can we then put our students in the place of the algebraic doctor, wire chief or salesman and

make them feel that the work of factoring proceeds along the same track as the mental work in the operations of real life? Can we stimulate them to the analysis of a polynomial by making them feel that they are thus living out in imagination at least their favorite occupation?

To come at length to the scheme for factoring which I suggested at first as being effective. The boy or girl must be taught a catalogue or list of algebraic symptoms or styles and the appropriate action associated with each name in this list. And this catalogue to reduce the labor of diagnosis to a minimum and yet make it certain must be logical, simple and catchy or easily remembered.

CATALOGUE OR SCHEME FOR FACTORING.

I. *Monomial Factor.*
$aQ + bQ - cQ.$

II. *2-terms (or Binomials).*
$Q^2 - O^2$
$Q^3 \pm O^3$
$Q^5 \pm O^5$

III. *3-terms (or Trinomials).*
$Q^2 \pm 2QO + O^2$
$Q^2 + (a + b)Q + ab$
$aQ^2 + bQ + c$
$Q^4 \pm mQ^2O^2 + O^4$

IV. *4-terms.*
$aQ + bQ + aO + bO$
$Q^2 + 2QO + O^2 - a^2$

V. *Factor Theorem.*

If we let a big round Q stand for a quantity then if we propose to look for a monomial factor we should expect to find a Q in each term and what would be a better name for this kind in our catalogue than "$aQ + bQ - cQ$"? If we frankly call it by this name we thus give it distinction and at the same time suggest the course to be followed in factoring it.

If we wish next to characterize the style with two squares and a minus sign we say this is $Q^2 - O^2$. We may notice at

this point that the Q and O were originally parentheses which have closed at top and bottom and thus acquired simpler and better known names, *e. g.*,

$$(a+b)^2-(c+d)^2=\underline{|a+b|}^2-\underline{|c+d|}^2.$$

And so on through the list of types which is here before us. They are arranged in order of difficulty and if a student tries the cases in this order he will generally obtain more quickly the correct diagnosis even if he has to discard two or three of the early trials. I have added the factor theorem as a last resort when the regular types fail though it is rather a luxury than a necessity.

It's a great scheme and its essentials are: (1) using Q and O and (2) trying each case in the order given, the former being the basis of its claim to novelty and effectiveness. The order is logical, but not necessarily pedagogical and the different cases can be taken up in any convenient order so long as the whole scheme is in the mind of the teacher and completed as a summary at the end. If your text develops the subject in this order well and good or if you take liberties with your text as an expert is entitled to you will find the order not only logical but simple, striking and easily remembered. Most books leave the teacher free to draw up a summary for himself. So if you refer to this list as the catalogue or menu and make your students feel that in this algebraic diagnosis they are being trained for the more vital sizing up of conditions in real life it will help to fasten the matter in the mind of the youthful and imaginative factorizer and tend to give his brain speed and his hand skill to perform the few operations that are so simply done when once decided on correctly.

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